OR-Neuron Based Hopfield Neural Network for Solving Economic Load Dispatch Problem

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Abstract — In this paper, we present stability analysis for an OR-neuron based Hopfield neural network. We explore the possibility of implementing higher order Hopfield neural network for solving economic load dispatch problem subjected to equality and inequality constraints. Previous work on this kind of problems is limited to quadratic cost function. The proposed approach is useful for higher order cost function as well. We solved and discussed illustrative examples in order to depict the usefulness of the proposed method.

Keywords — Hopfield neural network, OR-neuron, economic load dispatch, Lyapunov stability analysis

1. Introduction

The conventional Hopfield neural network model is the most commonly used model for auto-association and optimization. Hopfield networks are auto-associators in which node values are iteratively updated on local computation principle: the new state of each node depends only on its net weighted input at a given time [1,2]. This network is fully connected network and the weight matrix determination is one of the important tasks while using for certain applications.

In Hopfield neural network model, processing devices are called neurons. Each neuron has two states similar to those of McCullough and Pitts [4] firing 1 and not firing 0. Each neuron is connected to other neuron by a weight factor $W_{ij}$. In Hopfield’s basic model author assumed that there is no self connection and weights are symmetric, i.e., $W_{ij} = W_{ji}$. The stability of the model is demonstrated by giving a valid Lyapunov function or energy function. The alteration in states of Hopfield model causes monotonic decrement in energy or Lyapunov function. Further in the year 1984 [3], continuous architecture of the model is implemented using passive and active elements such as resistors, capacitors, and operational amplifiers. Considering nonlinear feature of operational amplifiers in terms of sigmoid function, author has demonstrated convergence of proposed Lyapunov energy function. Application of Hopfield neural network in optimization problems is demonstrated in [5], by solving the well defined Traveling-salesman problem. This application of Hopfield neural network in optimization problem has enabled its use in various fields. In the problem of optimization, the Hopfield neural network has a well demonstrated capability of finding solutions to difficult optimization problems. Researchers have applied the Hopfield neural network to solve Analog to Digital (A/D) conversion [6,7], Quadratic assignment problems [9], Job scheduling problems [8] etc. Besides these applications, Hopfield neural networks have been used in several other applications such as parametric identification of dynamical systems [12], economic load dispatch [11], and for solving a set of simultaneous linear equations [10].

Previous applications of Hopfield neural network was limited to quadratic cost function based optimization problems. In this paper, we propose an OR-neuron based Hopfield neural network which can be used to solve optimization problems with higher order cost functions. The stability analysis for the proposed model is carried out. Usefulness of proposed network is tested on the economic load dispatch problem with higher order cost function. A Lyapunov energy function for proposed network is formulated for solving economic load dispatch problem having cubic cost function. This energy function is used to determine weights and bias values of proposed network. Results obtained with proposed network are compared with conventionally used numerical
method, i.e., Newton’s method [13]. It has been shown that the proposed approach overcomes some of the important drawbacks of conventional numerical method while solving economic load dispatch problem, such as, calculation of Hessian matrix and its inverse.

The organization of this paper is as follows. In section 2, we present details of OR-neuron based Hopfield neural network and its stability analysis. We review mapping of conventional Hopfield neural network for solving quadratic cost function based economic load dispatch problems in section 3. Design details with proposed network for solving cubic cost function based economic load dispatch problem are demonstrated in section 4. Simulation results and discussion are presented in section 5. We concluded our work in section 6.

2. OR-Neuron Based Hopfield Neural Network

Conventional Hopfield neural network applications are limited to second order energy functions. In this section, an OR-neuron based Hopfield neural network is proposed for extending its applicability to higher order energy functions. We have used OR-equations as shown in Equation (1) for the development of OR-neuron based Hopfield neural network. The expressions for two input OR-neuron function is given by:

\[
\begin{align*}
    f(x_1) &= x_2 + x_3 - x_2x_3 \\
    f(x_2) &= x_1 + x_3 - x_1x_3 \\
    f(x_3) &= x_1 + x_2 - x_1x_2
\end{align*}
\]

Equation (1) is written in dynamical form by associating weights and biases to it. This is represented by:

\[
\begin{align*}
    \frac{df(x_1)}{dt} &= w_{12}x_2 + w_{13}x_3 - w_{123}x_2x_3 + I_1 \\
    \frac{df(x_2)}{dt} &= w_{21}x_1 + w_{23}x_3 - w_{213}x_1x_3 + I_2 \\
    \frac{df(x_3)}{dt} &= w_{31}x_1 + w_{32}x_2 - w_{312}x_1x_2 + I_3
\end{align*}
\]

Equation (2) is used to formulate OR-neuron based Hopfield neural network, the dynamics of the proposed model is depicted in Equation (3). Figure 1 shows the architecture of proposed model. It is observed from Equation (3) that it comprises of weighted linear and nonlinear terms. It is assumed that there is no self connection and weights are symmetric. States \( v_i \) are passed through the nonlinear function \( \phi(v_i) \) which results in \( x_i \) as its output. Output \( x_i \) is feedback as the input to neurons, and a kind of recurrence is achieved.

\[
\begin{align*}
    C_1 \frac{dv_1}{dt} &= -\frac{v_1}{R_1} + w_{12}x_2 + w_{13}x_3 - w_{123}x_2x_3 + I_1 \\
    C_2 \frac{dv_2}{dt} &= -\frac{v_2}{R_2} + w_{21}x_1 + w_{23}x_3 - w_{213}x_1x_3 + I_2 \\
    C_3 \frac{dv_3}{dt} &= -\frac{v_3}{R_3} + w_{31}x_1 + w_{32}x_2 - w_{312}x_1x_2 + I_3
\end{align*}
\]

Equation (3) is used to formulate a Lyapunov function which is used to carryout stability analysis of the proposed network. Equation 5 shows the expressions of energy for respective neurons, i.e., \( E_1, E_2, \) and \( E_3 \)

\[
\begin{align*}
    E_1 &= \left(-\frac{v_1}{R_1} + w_{12}x_2 + w_{13}x_3 - w_{123}x_2x_3 + I_1\right)x_1 \\
    E_2 &= \left(-\frac{v_2}{R_2} + w_{21}x_1 + w_{23}x_3 - w_{213}x_1x_3 + I_2\right)x_2 \\
    E_3 &= \left(-\frac{v_3}{R_3} + w_{31}x_1 + w_{32}x_2 - w_{312}x_1x_2 + I_3\right)x_3 \\
    E &= -(E_1 + E_2 + E_3)
\end{align*}
\]
Equation (4) is rewritten by combining the terms of identical order in:

\[ E = \left( \frac{v_1 x_1}{R_1} + \frac{v_2 x_2}{R_2} + \frac{v_3 x_3}{R_3} \right) - \left( w_{12} x_1 x_2 + w_{13} x_1 x_3 + w_{21} x_1 x_2 + w_{23} x_3 x_2 + w_{31} x_1 x_3 + w_{32} x_2 x_3 \right) \]

\[ + \left( w_{123} x_1 x_2 x_3 + w_{213} x_1 x_2 x_3 + w_{312} x_1 x_2 x_3 \right) - \left( I_1 x_1 + I_2 x_2 + I_3 x_3 \right) \] (5)

For proving the stability and convergence of proposed Lyapunov or energy function, following assumptions are taken into account:

1) No self connection: The condition for ignoring the self connections gives us following relations:

\[ w_{ii} = 0 \]

\[ w_{ii} = 0 \]

2) The weights are symmetric: After applying the symmetric condition we have following relations:

\[ w_{12} = w_{21} \]

\[ w_{13} = w_{31} \]

\[ w_{32} = w_{23} \]

\[ w_{123} = w_{312} = w_{213} \]

Generalized formulation of energy function for proposed network after considering the above mentioned assumptions is shown in Equation (9).

\[ E = -\frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} w_{ij} x_i x_j - \frac{1}{3} \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} w_{ijk} x_i x_j x_k + \sum_{i=1}^{3} I_i x_i \] (6)

States \( v_i \) of network are passed through the sigmoid nonlinearity, resulting in new states \( x_i = \varphi(v_i) \). This nonlinearity is strictly monotonously increasing and continuously differentiable function. Figure 2 depicts the essential properties of sigmoid non-linear function. Invertible property of sigmoid nonlinearity is used in Equation (6) for establishing the conditions of stability for proposed energy function. This results in:

\[ E = -\frac{1}{2} \sum_{i=1}^{3} \int_{0}^{\varphi^{-1}(x_i)} \frac{d\varphi(x_i)}{R_i} - \frac{1}{3} \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} w_{ijk} x_i x_j x_k + \sum_{i=1}^{3} I_i x_i \] (7)
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The stability of proposed energy function is speculated by calculating the derivative of energy function $E$ with respect to time. If $dE/dt \leq 0$ the proposed energy function is stable in Lyapunov sense. The derivative $dE/dt$ is represented by:

$$
\frac{dE}{dt} = -\frac{d}{dt} \left( -\sum_{i=0}^{1} v_i \frac{\phi^{-1}(x_i)}{R_i} dx_i \right) + \frac{d}{dt} \left( \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} w_{ij} x_i x_j \right) - \frac{d}{dt} \left( \frac{1}{3} \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} w_{ijk} x_i x_j x_k \right) + \frac{d}{dt} \left( \sum_{i=1}^{3} I_i \right)
$$

(8)

Equation (8) is represented in simplified manner by:

$$
\frac{dE}{dt} = \sum_{i=1}^{3} \frac{dE}{dx_i} \frac{dx_i}{dt}
$$

(9)

The derivative $dE/dx_i$ in Equation (9) is replaced with the terms given in equation (3) results in:

$$
\frac{dE}{dt} = \sum_{i=1}^{3} C_i \left( \frac{dv_i}{dx_i} \frac{dx_i}{dt} \right) = \sum_{i=1}^{3} C_i \left( \frac{d\phi^{-1}(x_i)}{dx_i} \left( \frac{dx_i}{dt} \right)^2 \right)
$$

(10)

Following observations are drawn from above equations:

1) The term $d\phi^{-1}(x_i)/dx_i$ is always positive, as $\phi^{-1}(x_i)$ is strictly monotonically increasing, invertible, and continuously differentiable function.
2) The term $(dx_i/dt)^2$ is a quadratic term and is positive for all real values.

It is evident from the above observations that the $dE/dt$ (Equation 8) always remain less than or equal to zero. Hence it can be stated that the proposed Lyapunov energy function leads to global minima and the dynamics of the proposed model is stable.
Generalized OR-Neuron Based Hopfield Neural Network:

The above derivation is limited to two input OR-neuron based Hopfield neural network. In this subsection, we present the above analysis for generalized OR-neuron based Hopfield neural network.

Equation (11) shows the dynamical equations for generalized OR-neuron based Hopfield neural network. In order to prove its stability following assumptions are made:

1) The synaptic weights are symmetric, i.e., \( w_{ij} = w_{ji} \), \( w_{ijk} = w_{kij} = w_{kji} \), \( w_{ijkl} = w_{jkil} = w_{kijl} = w_{lkij} \).
2) There is no self connections, i.e., \( w_{ii} = 0 \), \( w_{iii} = 0 \), \( w_{iiii} = 0 \).

\[
\frac{du_j}{dt} = -\left( -\frac{V_j}{R_j} + \frac{1}{2} \sum_{j=1}^{n} w_{ij} x_j - \frac{1}{3} \sum_{j=1}^{n} \sum_{k=1}^{n} w_{ijk} x_j x_k + \frac{1}{4} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} w_{iklj} x_j x_k x_l + \ldots + I_i \right) \tag{11}
\]

The energy function for generalized OR-based Hopfield neural network is shown in equation (12). The stability analysis for generalized model can be proved in the similar manner as we did above. It can be stated that \( \frac{dE}{dt} \) for equation (18) is always less than or equal to zero.

\[
E = \left( -\sum_{i=1}^{n} \frac{V_i x_i}{R_i} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j - \frac{1}{3} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} w_{ijk} x_i x_j x_k + \frac{1}{4} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} w_{iklj} x_i x_j x_k x_l + \ldots + \sum_{i=1}^{n} I_i x_i \right) \tag{12}
\]

The generalized OR-based Hopfield neural network can be used for solving higher order optimization problems. For this, we have to formulate energy function in such a way that it maps the higher order optimization problem. This mapping provides expression for weights and biases in terms of coefficients of higher order cost function. Substituting these weights and biases in the dynamical equations of network and simulating it by Euler’s method or any other numerical methods provide the desired solutions for the optimization problem.

3. Hopfield Neural Network for Quadratic Cost Function

We first review the mapping of conventional Hopfield neural network to the quadratic cost function \( C_i \) (Equation 13).

\[
C_i = \sum_i \left( a_i + b_i P_i + c_i P_i^2 \right) \tag{13}
\]

where \( C_i \) is total cost, \( a_i, b_i, c_i \) are constant coefficients, and \( P_i \) is the generated power of generator \( i \). In minimizing total cost, following constraints should be satisfied

1) Power balance constraints: \( D + L = \sum_i P_i \)

where \( D \) is total demand and \( L \) is power loss which is represented as \( L = \sum_i \sum_j a_{ij} P_i P_j \), \( a_{ij} \) are transmission line loss coefficients.

2) Generation limits constraint, each unit has its maximum and minimum generation limits, i.e.

\[
P_{i} \leq P_i \leq \overline{P}_{i}
\]

where \( P_{i} \) and \( \overline{P}_{i} \) are minimum and maximum generation limits of unit \( i \) respectively.

The energy function for solving the economic load dispatch problem having quadratic cost function is defined in Equation (14) and (15).

\[
E = \frac{A \left( D + L - \sum_i P_i \right)^2}{2} + \frac{B \sum_i \left( a_i + b_i P_i + c_i P_i^2 - \sum_j P_j \right)}{2} \tag{14}
\]
where \( A \geq 0 \) and \( B \geq 0 \) are weighing factors. Synaptic strengths \( W_{ij} \) and bias values \( I_i \) of network are obtained by mapping the above energy function with conventional Hopfield neural network energy function [11]. Expression for connectionist weights and bias for conventional Hopfield neural network are given by:

\[
\sum_{j} A P_j P_{ij} + B \sum_{i} a_i = \frac{A(D + L)}{2} - \sum_{i} \left( A(D + L) + \frac{Bb_i}{2} \right) P_i + \sum_{i} \sum_{j} (A + Bc_i) \frac{P_i P_j}{2}
\]

Expression for connectionist weights and bias for conventional Hopfield neural network are given by:

\[
dU_i = \sum_{j} T_{ij} v_j + I_i
\]

where \( T_{ij} = -A - Bc_j \)

\[
T_{ij} = -A
\]

\[
I_i = A(D + L) - \frac{Bb_i}{2}
\]

The differential synchronous transition model used in computation for this Hopfield neural network is given by:

\[
U_i(k + 1) = U_i(k) + \Delta t \left( \sum_{j} T_{ij} v_j(k) + I_i \right)
\]

\[
v_j(k) = \phi \left( U_i(k) \right)
\]

In order to incorporate generation limits constraints following nonlinear function has been used:

\[
v_j(k) = \phi \left( U_i(k) \right) = \frac{P_i - P_j}{1 + e^{V_{i,j}/\theta}} + P_j
\]

This methodology is suitable for those economic load dispatch problems with quadratic cost function. For economic load dispatch problems having higher cost function, this methodology is inferior. To overcome this limitation of conventional Hopfield neural network, in next section we present OR-neuron based Hopfield neural network approach for solving higher order cost function based economic load dispatch problems.

4. Proposed Hopfield Neural Network for Cubic Cost function Problem

Generator characteristics in a system can be of any order polynomial; more the order of polynomial more accurately it maps the characteristics of generator. In this application we considered generators whose characteristics are described by cubic polynomial. In this section, we present OR-neuron based Hopfield neural network approach for solving cubic cost function based economic load dispatch problems. The cubic cost function of generator is given by:

\[
C_i = \sum_{i} \left( a_i + b_i P_i + c_i P_i^2 + d_i P_i^3 \right)
\]

where \( C_i \) is total cost, \( a_i, b_i, c_i, d_i \) are constant coefficients, and \( P_i \) is the generated power of generator \( i \). This problem also considered above mentioned two constraints, i.e., Power balance constraints and Generation limits constraints. We have defined energy function in Equation (20) which is used for solving the cubic cost function based economic load dispatch problem.

\[
E = \frac{A(D + L)}{2} - \sum_{i} \left( A(D + L) + \frac{Bb_i}{2} \right) P_i^2 + \sum_{i} \sum_{j} (A + Bc_i) \frac{P_i P_j}{2} - \sum_{i} P_i
\]
\[
E = \frac{A(D + L)^2}{2} - \sum_i \left( A(D + L) + \frac{Bb_i}{2} \right) P_i + \sum_i \sum_j (A + Bc_i) \frac{PP_iP_j}{2} + \sum_i \sum_{j \neq k} \frac{PP_iP_j}{2} + B \sum_i \frac{d_i}{2} + Bd_i \sum_i \left( \sum_j \frac{PP_iP_j}{2} \right)
\]

where \( A \geq 0 \) and \( B \geq 0 \) are weighing factors. By comparing coefficients of both equations (Equation 7 and 21) we derived expressions for bias \( I_i \), first order weights \( W_{ij} \), and higher order weights \( W_{ijk} \). These weights and bias values are used in formulation of OR-neuron based Hopfield neural network equation. This is given by:

\[
\frac{dU_i}{dt} = \sum_j \sum_k T_{ijk}v_j v_k + \sum_j T_{ij}v_j + I_i
\]

The expression for weights and biases are given by:

\[
T_{ij} = -A - Bc_i \\
T_{ij} = -A \\
T_{ijk} = -Bd_i, \text{ at } i = j = k \\
T_{ijk} = 0, \text{ at } i \neq j \neq k \\
I_i = A(D + L) - \frac{Bb_i}{2}
\]

With these weights and biases the OR-neuron based Hopfield neural network is simulated using Euler’s method for solving cubic cost function based economic load dispatch problem. Expression of differential synchronous transition model used in computation for the OR-neuron based Hopfield neural network is given by:

\[
U_i(m + 1) = U_i(m) + \Delta t \left( \sum_j \sum_k T_{ijk}v_j(m)v_k(m) + \sum_j T_{ij}v_j(m) + I_i \right)
\]

\[
v_j(m) = \varphi_j(U_j(m))
\]

Generation limits constraints are incorporated by:

\[
v_j(m) = \varphi_j(U_j(m)) = \frac{1}{1 + e^{-U_j/\alpha_j}} + P_i
\]

where \( \alpha_j \) is the logistic gain, which decides the spread of the sigmoid function.

### 5. Simulation Results and Discussion

The usefulness of proposed model is tested on two different examples given in [13]. Simulation results with proposed approach are compared to results obtained with Newton’s method [13] on cubic cost function economic load dispatch problems.

#### A. Example 1

We wish to determine the economic operating point for these three units when delivering a total of 2500 MW. The input-output characteristics of generating units are given by:

\[
H(MBtu/h) = a_i + b_iP_i + c_iP_i^2 + d_iP_i^3 \quad (P \text{ in MW})
\]

Table 1 shows the values for the coefficient corresponding to three generating units.

<table>
<thead>
<tr>
<th>Units (i)</th>
<th>( a_i )</th>
<th>( b_i )</th>
<th>( c_i )</th>
<th>( d_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>749.55</td>
<td>6.95</td>
<td>9.68 \times 10^4</td>
<td>1.27 \times 10^7</td>
</tr>
<tr>
<td>Unit 2</td>
<td>1285.0</td>
<td>7.051</td>
<td>7.375 \times 10^4</td>
<td>6.453 \times 10^8</td>
</tr>
<tr>
<td>Unit 3</td>
<td>1531.0</td>
<td>6.531</td>
<td>1.04 \times 10^4</td>
<td>9.98 \times 10^8</td>
</tr>
</tbody>
</table>
The fuel cost is 1.0 R/MBtu for each unit and unit limits as follows:

\[
\begin{align*}
320\text{MW} & \leq P_1 \leq 800\text{MW} \\
300\text{MW} & \leq P_2 \leq 1200\text{MW} \\
275\text{MW} & \leq P_3 \leq 1100\text{MW}
\end{align*}
\]

Weights and bias values for the network are derived by using expressions given in Equation (23). The Euler’s formation of the network dynamics (Equation 24) are simulated numerically for achieving the desired results. The essential parameters required for simulation are given in Table 1. In this example we neglected the system loss, i.e., \( P_L = 0 \). The proposed network uses two penalty factors A and B. A is associated with the constraint of total load demand, and B is the penalty factor to the constraint of objective function. It is found that when A was bigger than 0.6 regardless of B values, the network shows oscillations. Selection of appropriate values for A and B during simulation is one of the limitations. Results obtained from proposed and conventional method are shown in Table 2. It is observed that there exists a small mismatch in values of power but it can be ignored because the generators are operating within the constraints level. It is also evident from Table 2 that the total fuel costs obtained with both the methods are almost equal. Energy profile for the network during simulation is plotted in Figure 3. The variation in power of each generator is drawn in Figure 4. It is observed from these figures that the network reaches to a desired solution in less iterations.

### Table 2. Simulation results for Example 1

<table>
<thead>
<tr>
<th>Method</th>
<th>( P_1 ) (MW)</th>
<th>( P_2 ) (MW)</th>
<th>( P_3 ) (MW)</th>
<th>Total Power (MW)</th>
<th>Total Cost ($/h)</th>
<th>Total Simulation Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hopfield Net Method</td>
<td>712.4</td>
<td>1176.4</td>
<td>609.9</td>
<td>2498.6</td>
<td>22863.00</td>
<td>1.656</td>
</tr>
<tr>
<td>Newton’s Method</td>
<td>724.99</td>
<td>910.15</td>
<td>864.85</td>
<td>2500.0</td>
<td>22729.00</td>
<td>3.265</td>
</tr>
</tbody>
</table>

Figure 3. Energy profile for the proposed network for \( P_L = 0 \).

Figure 4. Variation in generators power, i.e., \( P_1, P_2, \) and \( P_3 \) with respect to iterations under \( P_L = 0 \).

### B. Example 2

In this example we incorporated the network losses as one of the constraint. In Example 1 this constraint was assumed as zero. The losses in system are given by Equation (27). The problem for finding economic operation of generating units is solved by proposed approach. Results are compared to conventional Newton’s method. Table 3 presents the details of simulation results obtained from both the approaches. It is observed from table that there exists a negligibly small mismatch in the values of total cost which can be ignored. The energy profile for the proposed network is shown in Figure 5. The variation in power of each generator, i.e., \( P_1, P_2, \) and \( P_3 \) are shown in Figure 6.

\[
P_k = 3.0 \times 10^{-5} P_1^2 + 9.0 \times 10^{-5} P_2^2 + 12.0 \times 10^{-5} P_3^2
\]  

(27)
### Table 3. Simulation Results for Example 2

<table>
<thead>
<tr>
<th>Method</th>
<th>$P_1$ (MW)</th>
<th>$P_2$ (MW)</th>
<th>$P_3$ (MW)</th>
<th>Total Power (MW)</th>
<th>Total Cost ($/h$)</th>
<th>Total Simulation Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hopfield Net Method</td>
<td>731.0</td>
<td>1000.9</td>
<td>908.9</td>
<td>2640.8</td>
<td>23942.00</td>
<td>3.843</td>
</tr>
<tr>
<td>Newton’s Method</td>
<td>1053.7</td>
<td>868.3</td>
<td>746.0</td>
<td>2667.9</td>
<td>24322.5</td>
<td>3.969</td>
</tr>
</tbody>
</table>

It is evident from the results obtained in Example 1 and Example 2 that the proposed method provides a comparable way to solve difficult economic load dispatch problem. Previous application of Hopfield neural network in economic load dispatch problem was limited for quadratic cost functions only. Whereas, the methodology proposed in this paper is applicable to economic load dispatch problems with higher order cost functions. The results obtained with proposed approach are compared to one of the method given in [13], i.e., Newton’s method. Newton’s method requires the calculation of Hessian matrix and its inverse, which requires larger computations which is one of the limitations. The proposed network works on the principle of energy minimization and calculation of Hessian matrix and its inverse are not needed. This saves a lot of computations. Proposed network works on fixed weights and hence the simulation time required is also less as compared with the conventional approaches. The determination of weights and biases values for the network is one of the important tasks. The selection of values for penalty factor is one of the limitations of present approach.

![Energy profile for the proposed network](image1)

![Variation in generators power](image2)

### 6. Conclusion

In this paper, OR-neuron based Hopfield neural network is proposed for the higher order cost function based economic dispatch problems. A new energy function is proposed which incorporates imposed constraints. The proposed method has been tested on a 3-unit system having cubic cost function. Results obtained with proposed approach are compared with Newton’s method. It is observed that the proposed method requires less number of iterations to achieve convergence. It is also found that the proposed method overcomes the drawback of calculating Hessian matrix and its inverse in Newton’s method, which creates problem when the matrix becomes singular. Proposed method is extendable for solving any order cost function based optimization problem and it takes care of imposed constraints. Simulation results shows that the solutions obtained from the proposed method are fast and accurate.

### References

OR-Neuron Hopfield NN for Economic Load Dispatch Problem

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